On measuring the masses of pair-produced semi-invisibly decaying particles at hadron colliders

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ABSTRACT: A straightforward new technique is introduced which enables measurement at hadron colliders of an analytical combination of the masses of pair-produced semi-invisibly decaying particles and their invisible decay products. The new technique makes use of the invariance under contra-linear Lorentz boosts of a simple combination of the transverse momentum components of the aggregate visible products of each decay chain. In the general case where the invariant masses of the visible decay products are non-zero it is shown that in principle the masses of both the initial particles from the hard scattering and the invisible particles produced in the decay chains can be determined independently. This application is likely to be difficult to realise in practice however due to the contamination of the final state with ISR jets. The technique may be of most use for measurements of SUSY particle masses at the LHC, however the technique should be applicable to any class of hadron collider events in which heavy particles of unknown mass are pair-produced and decay to semi-invisible final states.

KEYWORDS: Hadron-Hadron Scattering.

Contents

1. Introduction

In R-Parity conserving SUSY events at hadron colliders SUSY particles ('sparticles') must be pair-produced and undergo cascade decay to the Lightest Supersymmetric Particle (LSP), which is often invisible and hence a dark matter candidate. The presence of two such invisible particles in the final state, together with imperfect detector hermeticity close to the beam-pipe and an uncertain parton centre-of-mass energy, prevents the use of conventional invariant mass or transverse mass techniques for sparticle mass measurement. Similar challenges are faced when attempting to measure the mass of any pair-produced particles with visible and invisible decay products.

Several approaches to this general problem have been documented, usually in the context of measuring SUSY particle masses. Given a sufficiently long decay chain constraints on analytical combinations of sparticle masses can be obtained from the positions of endpoints in distributions of invariant masses of combinations of visible SUSY decay products (jets, leptons etc.) [1]. Given a number of such constraints, the system of equations may be solved with a numerical fit to obtain the individual masses [1–3]. It was recently shown that the mass precision obtained from this technique can be improved by subsequently performing combined fits to individual events, imposing both experiment end-point constraints and event E_T^{miss} constraints [4].

When the number of kinematic end-point constraints provided by a given decay chain is insufficient to fully constrain individual sparticle masses alternative techniques must be employed. One possible approach involves solving simultaneously the mass-shell conditions obtained from several events containing the same decay chain [5]. This *mass-relation method* exploits the small widths of SUSY states, allowing the mass of each state appearing in the considered events to be assumed to be constant.

A second approach to this problem is to select events in which the same decay chain appears in both 'legs' of each selected event. In this case additional constraints are provided by the components of the event $\mathbf{E_T}^{\text{miss}}$ vector,[1] and again use can be made of the sparticle narrow-width approximation to equate the masses in the two legs. This permits the construction of further distributions with kinematic end-points related to the masses of sparticles present in the event.

One example of a technique of this kind is the *stransverse mass* method [6, 2, 7]. Consider two identical heavy SUSY states δ_1 and δ_2, decaying respectively to visible products v_1 and v_2 and identical lighter states α_1 and α_2. If $\mathbf{p_T}(\alpha_1)$, the transverse momentum vector of α_1, were known then it would be possible to calculate $m_T(\delta_1)$, the transverse mass of δ_1, which is bounded from above by $m(\delta)$. If $\mathbf{p_T}(\alpha_1)$ is known however then so is $\mathbf{p_T}(\alpha_2)$ through application of the event $\mathbf{E_T}^{\text{miss}}$ constraints. Therefore $m_T(\delta_2)$ could also be calculated, a quantity which must also be less than $m(\delta)$. Consequently the maximum value of $m_T(\delta_1)$ and $m_T(\delta_2)$ provides a variable with an end-point whose position measures $m(\delta)$. Of course in reality we are not able to measure $\mathbf{p_T}(\alpha_1)$ or $\mathbf{p_T}(\alpha_2)$ however the great insight of ref. [6] was the realisation that if we can find a test value $\mathbf{p_T}(\alpha_1)$ which minimises this maximum transverse mass, we can be sure that the minimised-maximised transverse mass is also bounded from above by $m(\delta)$. This 'minimax' transverse mass quantity is referred to as the 'stransverse mass' or M_{T2}.

The development of the stransverse mass technique was particularly important because for the first time it allowed the measurement of masses of sparticles decaying through very short cascades, for instance $\tilde{q}_R \to q\tilde{\chi}_1^0$ or $\tilde{l} \to l\tilde{\chi}_1^0$. Furthermore an analytical expression for M_{T2} has recently been derived, valid in cases where the centre-of-mass (CoM) frame is at rest in the laboratory transverse plane [8], thus simplifying its use considerably. The technique inherits one draw-back from its use of the transverse masses of δ decay products however, namely that it requires the use of $m(\alpha)$ as an input. M_{T2} may therefore be described more correctly as an ensemble of variables, one for each assumed value for the unknown quantity $m(\alpha)$. The dependence of M_{T2} on $m(\alpha)$ has been determined to be approximately $m(\delta) - m(\alpha)$ in specific cases [7], however it would in general be preferable if the definition of the variable were independent of the unknown quantities to be measured. In that case the mass constraints obtained from an end-point fit would be uncorrelated with other measurements and hence could be used as input to a global mass fit.

In this paper we will propose a very simple technique which seeks to address the same problem as the stransverse mass technique, but which approaches the problem from a different perspective. The new technique will allow a simple analytical combination of particle/sparticle masses to be constrained in a precise and model-independent manner. Furthermore the technique will offer at least in principle the prospect of measuring individual particle masses, as postulated for the stransverse mass technique in refs. [9 – 12]. The new technique will be applicable to any class of events in which heavy particles of unknown mass are pair-produced and decay to semi-invisible final states.

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[1]We denote three-vector and two-vector quantities with **bold** case, while the corresponding magnitudes are denoted with standard case. Four-vector quantities are written in standard case.

The structure of the paper is as follows. Section 2 will describe the principles underlying the technique and investigate the properties of the new variable upon which it is based. Section 3 will illustrate application of the technique to the problem of constraining sparticle mass combinations with $\tilde{q}_R \to q\tilde{\chi}_1^0$ pair events at the LHC. Section 4 will outline extension of the technique to measurement of individual particle masses. Section 5 will conclude and discuss avenues for future work.

2. Description of technique

2.1 Background

Consider 'symmetric' events in which identical cascade decay chains of the form

$$\delta \to \alpha v \tag{2.1}$$

occur in each leg i of the event. We shall refer to the initial particles produced in the hard scattering as δ_i. We shall further consider n step decay chains in each leg consisting of $n-1$ decays, such that the $(n-1)^{\text{th}}$ decays produce invisible particles α_i. The visible products of decays 1 to $(n-1)$ in each leg will be considered as single systems v_i of mass $m(v_i)$ and four-momentum $p(v_i)$. We shall assume that no invisible particles other than α are produced in the decay chains. The particles δ_i and α_i have common masses which are respectively $m(\delta)$ and $m(\alpha)$.

This parameterisation of the decay chains is quite general. The case $n = 2$ corresponds to SUSY chains such as $\tilde{q}_R \to q\tilde{\chi}_1^0$ or $\tilde{l} \to l\tilde{\chi}_1^0$, with α identified as the LSP $\tilde{\chi}_1^0$. In these cases $m^2(v_i) \ll p^2(v_i)$ at the LHC. For longer SUSY chains we can choose the number of decays provided we can unambiguously identify the visible products of those decays. If n is equal to the total number of sparticles in the chain then α is again the LSP. For chains with $n > 2$ steps the distributions of invariant masses $m(v_i)$ can display kinematic end-points sensitive to analytical combinations of sparticle masses appearing in the chain [1]. This information is used to constrain the individual masses in the end-point method but will be incidental to the technique described here.

Consider now the use of N symmetric events of the general form of eq. 2.1 to measure $m(\delta)$ and $m(\alpha)$. This problem reduces to one of solving $6N$ non-linear simultaneous equations, with each event providing the mass-shell conditions:

$$[p(v_1) + p(\alpha_1)]^2 = [p(v_2) + p(\alpha_2)]^2 = m^2(\delta),$$
$$[p(\alpha_1)]^2 = [p(\alpha_2)]^2 = m^2(\alpha), \tag{2.2}$$

together with two $\mathbf{E_T}^{\text{miss}}$ constraints:

$$p_x(\alpha_1) + p_x(\alpha_2) = E_x^{\text{miss}},$$
$$p_y(\alpha_1) + p_y(\alpha_2) = E_y^{\text{miss}}. \tag{2.3}$$

Each event contributes 2 unknown masses, which are common to all events, and 8 unknown α_i four-momentum components, which differ between events. The total number of unknown

parameters is therefore $8N + 2$ while the number of constraints is $6N$ and so the system of equations is highly under-constrained.

It may seem surprising at first that the above system of equations can be solved at all, however it should be noticed that we are not concerned with measuring the four-momenta $p(\alpha_1)$ and $p(\alpha_2)$ for all events, but rather with measuring only the common masses $m(\delta)$ and $m(\alpha)$ using at least one event. Consequently we may set out to discard events in which the unknown masses depend on unknown four-momentum components. The problem therefore reduces to one of finding variables dependent only on the measurable quantities $p(v_1)$, $p(v_2)$ and $\mathbf{E_T}^{\text{miss}}$ which identify events where the masses also depend only on those measurable quantities. This general approach is effectively that taken by kinematic end-point techniques, in which the variables identifying the events, such as $m(ll)$ or M_{T2}, are also those which provide the mass measurment. This is also the approach which shall be taken here.

2.2 Transverse momentum end-points

One possible starting point for this problem was outlined in ref. [13]. In an effective two-body decay process of the type considered above the magnitude of the three-momentum of the visible decay products in the rest frame of δ_i is given by

$$
\begin{aligned}
|\mathbf{p}(v_i)| &= \frac{1}{2} \frac{\sqrt{[m^2(\delta) - m^2(\alpha) + m^2(v_i)]^2 - [2m(\delta)m(v_i)]^2}}{m(\delta)} \\
&\equiv \frac{1}{2} \mathcal{M}_i,
\end{aligned}
\tag{2.4}
$$

which defines the 2-body mass parameter $\mathcal{M}_i$. It will also be useful for the discussion which follows to define the equivalent quantity $\mathcal{M}_0$ for the special case where $m(v_i) = 0$:

$$
\mathcal{M}_0 \equiv \frac{m^2(\delta) - m^2(\alpha)}{m(\delta)}.
\tag{2.5}
$$

If δ_i has a small boost in the laboratory transverse frame, then the laboratory transverse momentum of v_i is of order $\mathcal{M}_i/2$. This dependence of the momenta of visible decay products on the masses of heavy particles further up the decay chain is the reason that variables such as the 'effective mass' [1] used in SUSY studies are sensitive to such masses.

In principle we can improve on the use of *ad hoc* variables such as the effective mass however. In the rest frame of δ_i the magnitude of the momentum of v_i transverse to the beam direction,[2] $p_T(v_i)$, is bounded from above by $\mathcal{M}_i/2$ because

$$
p_T(v_i) = \frac{\mathcal{M}_i}{2} \sin \psi_i,
\tag{2.6}
$$

where ψ_i is the polar decay angle relative to the beam direction. Consequently if we could measure $p_T(v_i)$ we could constrain the masses.

Unfortunately however we are not able to measure $p_T(v_i)$ directly — instead we measure the equivalent quantity in the laboratory frame: $p'_T(v_i)$. To proceed further we assume

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[2]We denote quantities measured in the $\delta_1 \delta_2$ CoM frame with primed variables and those measured in the rest frames of δ_1 or δ_2 with unprimed variables.

that the $\delta_1\delta_2$ CoM frame is at rest in the laboratory transverse plane. This condition can be enforced by selecting events in which the net transverse momentum of the final state excluding the δ_1 and δ_2 decay products is small. In this case $p'_T(v_i)$ is related to $p_T(v_i)$ by a proper Lorentz transformation in the transverse plane through the well-known relation:

$$p'^2_T(v_i) = \frac{1}{1-\beta^2}\left[p_T(v_i)\cos\phi_i \pm \beta E(v_i)\right]^2 + p^2_T(v_i)\sin^2\phi_i, \qquad (2.7)$$

where β is the transverse boost factor $(0 < \beta < 1)$, $E(v_i)$ is the energy of v_i and ϕ_i is the angle in the rest frame of δ_i between the boost direction and $\mathbf{p_T}(v_i)$. For given v_i we know neither β nor ϕ_i and hence we are not able to reconstruct $p_T(v_i)$. Nevertheless we do know from conservation of momentum that in the $\delta_1\delta_2$ CoM frame, and hence the laboratory transverse plane, the boost applied to v_2 is equal and opposite to that applied to v_1.

To proceed further we shall attempt to find a quantity which can be calculated from the components of $\mathbf{p_T}(v_1)$ and $\mathbf{p_T}(v_2)$ which remains unchanged if calculated with the corresponding components of $\mathbf{p'_T}(v_1)$ and $\mathbf{p'_T}(v_2)$. If we could find such a quantity then we could use it to relate momenta measured in the $\delta_1\delta_2$ CoM frame to those measured in the δ_1 and δ_2 rest frames and hence constrain $\mathcal{M}_i$.

2.3 Cotransverse mass and contransverse mass

Consider first a system containing two particles v_1 and v_2 with masses $m(v_1)$ and $m(v_2)$ measured in some frame $F_{(0)}$ to have four-momenta $p(v_1)$ and $p(v_2)$. If both these particles are now measured in a different frame $F_{(1)}$ it is well known that the mass obtained from $p(v_1) + p(v_2)$ remains unchanged, i.e. the quantity

$$\begin{aligned}
m^2(v_1, v_2) &= [E(v_1) + E(v_2)]^2 - [\mathbf{p}(v_1) + \mathbf{p}(v_2)]^2 \\
&= m^2(v_1) + m^2(v_2) + 2[E(v_1)E(v_2) - \mathbf{p}(v_1)\cdot\mathbf{p}(v_2)]
\end{aligned} \qquad (2.8)$$

is invariant. Another way to interpret this is that when particles v_1 and v_2 are subjected to co-linear boosts of equal magnitude $m^2(v_1, v_2)$ is invariant.

Now let us examine what happens when we start from one frame $F_{(0)}$, but boost particles v_1 and v_2 to different frames $F_{(1)}$ and $F_{(2)}$ respectively. These new frames are distinguished by the fact that their boosts are of equal magnitude but *opposite* direction in frame $F_{(0)}$. In other words particles v_1 and v_2 are subjected to contra-linear boosts of equal magnitude. Clearly $m^2(v_1, v_2)$ is no longer an invariant — this can be seen for instance by considering $\mathbf{p}(v_1) = -\mathbf{p}(v_2)$ in which case $E(v_1) + E(v_2)$ increases with increasing β while $\mathbf{p}(v_1) + \mathbf{p}(v_2)$ remains zero.

Consider now a new quantity M_C equivalent to the invariant mass obtained from $p(v_1) + \mathcal{P}(p(v_2))$ where $\mathcal{P}$ is the standard parity transformation operator:

$$\begin{aligned}
M^2_C(v_1, v_2) &\equiv [E(v_1) + E(v_2)]^2 - [\mathbf{p}(v_1) - \mathbf{p}(v_2)]^2 \\
&= m^2(v_1) + m^2(v_2) + 2[E(v_1)E(v_2) + \mathbf{p}(v_1)\cdot\mathbf{p}(v_2)].
\end{aligned} \qquad (2.9)$$

This quantity is invariant under the contra-linear boosts considered above. Denoting quantities measured in $F_{(0)}$ with primed variables, and those measured in $F_{(1)}$ and $F_{(2)}$ with

unprimed variables, and defining the $\hat{x}$ direction to be the boost direction, this can easily be demonstrated:

$$
\begin{aligned}
M_C'^2(v_1, v_2) &= [E'(v_1) + E'(v_2)]^2 - [\mathbf{p}'(v_1) - \mathbf{p}'(v_2)]^2 \\
&= \gamma^2 [E(v_1) + \beta p_x(v_1) + E(v_2) - \beta p_x(v_2)]^2 \\
&\quad -\gamma^2 [p_x(v_1) + \beta E(v_1) - p_x(v_2) + \beta E(v_2)]^2 \\
&\quad - [p_y(v_1) - p_y(v_2)]^2 - [p_z(v_1) - p_z(v_2)]^2 \\
&= \gamma^2([E(v_1) + E(v_2)]^2 + \beta^2 [p_x(v_1) - p_x(v_2)]^2 \\
&\qquad + 2\beta[E(v_1) + E(v_2)][p_x(v_1) - p_x(v_2)]) \\
&\quad -\gamma^2(\beta^2 [E(v_1) + E(v_2)]^2 + [p_x(v_1) - p_x(v_2)]^2 \\
&\qquad + 2\beta[E(v_1) + E(v_2)][p_x(v_1) - p_x(v_2)]) \\
&\quad - [p_y(v_1) - p_y(v_2)]^2 - [p_z(v_1) - p_z(v_2)]^2 \\
&= \gamma^2 \left([E(v_1) + E(v_2)]^2[1 - \beta^2] - [p_x(v_1) - p_x(v_2)]^2[1 - \beta^2]\right) \\
&\quad - [p_y(v_1) - p_y(v_2)]^2 - [p_z(v_1) - p_z(v_2)]^2 \\
&= [E(v_1) + E(v_2)]^2 - [\mathbf{p}(v_1) - \mathbf{p}(v_2)]^2 \\
&= M_C^2(v_1, v_2).
\end{aligned} \tag{2.10}
$$

Since $M_C(v_1, v_2)$ is invariant under contra-linear boosts of equal magnitude its value can be calculated from the momenta and energies of v_1 and v_2 in any pair of frames $\mathrm{F}_{(1)}$ and $\mathrm{F}_{(2)}$ related to $\mathrm{F}_{(0)}$ by such boosts. For instance in the case considered above $\mathrm{F}_{(0)}$ could be identified with the $\delta_1 \delta_2$ CoM frame and $\mathrm{F}_{(1)}$ and $\mathrm{F}_{(2)}$ identified with the rest frames of δ_1 and δ_2, in which $|\mathbf{p}(v_1)| = \mathcal{M}_1/2$ and $|\mathbf{p}(v_2)| = \mathcal{M}_2/2$.

From a practical perspective the quantity $M_C(v_1, v_2)$ defined by eq. 2.9 is relevant only to cases where the $\delta_1 \delta_2$ CoM frame is at rest in the laboratory frame, for instance in collisions at a lepton collider such as LEP or the ILC. At a hadron collider the scenario is more complicated. As discussed above, co-linear boosts in the laboratory transverse plane can be limited by selecting events in which the net transverse momentum of the final state excluding the δ_1 and δ_2 decay products is small. There remains however a potentially large co-linear boost in the beam ($\hat{z}$) direction caused by the differing proton momentum fractions of the colliding partons in the event initial state. $M_C(v_1, v_2)$ is not invariant under co-linear boosts of v_1 and v_2 because $\mathcal{P}$ does not commute with proper Lorentz transformations. Consequently we must focus purely on quantities constructed from momentum components measured in the laboratory plane transverse to the beam direction.

If v_1 and v_2 were subjected to co-linear rather than contra-linear equal magnitude boosts in the laboratory transverse plane then a suitable invariant quantity to consider would be the transverse mass $m_T(v_1, v_2)$ [14], hereafter refered to as the *cotransverse mass*.

$m_T(v_1, v_2)$ is defined by:

$$m_T^2(v_1, v_2) = [E_T(v_1) + E_T(v_2)]^2 - [\mathbf{p_T}(v_1) + \mathbf{p_T}(v_2)]^2$$
$$= m^2(v_1) + m^2(v_2) + 2[E_T(v_1)E_T(v_2) - \mathbf{p_T}(v_1) \cdot \mathbf{p_T}(v_2)], \qquad (2.11)$$

where

$$E_T(v_i) = \sqrt{p_T^2(v_i) + m^2(v_i)}. \qquad (2.12)$$

This quantity is useful because it is bounded from above by $m(v_1, v_2)$. When $m(v_1) = m(v_2) = 0$ the following simplification can be made:

$$m_T^2(v_1, v_2) = 2p_T(v_1)p_T(v_2)(1 - \cos\phi_{12}), \qquad (2.13)$$

where ϕ_{12} is the angle between v_1 and v_2 in the transverse plane. This illustrates that events saturating the bound on the (co)transverse mass typically require that v_1 and v_2 be back-to-back.

In the case of contra-linear equal magnitude boosts considered above the equivalent quantity to the (co)transverse mass can be derived from eq. 2.9:

$$M_{\mathrm{CT}}^2(v_1, v_2) \equiv [E_T(v_1) + E_T(v_2)]^2 - [\mathbf{p_T}(v_1) - \mathbf{p_T}(v_2)]^2$$
$$= m^2(v_1) + m^2(v_2) + 2[E_T(v_1)E_T(v_2) + \mathbf{p_T}(v_1) \cdot \mathbf{p_T}(v_2)]. \qquad (2.14)$$

We shall refer to this quantity as the *contransverse mass*. This has the property that when $m(v_1) = m(v_2) = 0$ it reduces to

$$M_{\mathrm{CT}}^2(v_1, v_2) = 2p_T(v_1)p_T(v_2)(1 + \cos\phi_{12}), \qquad (2.15)$$

where if $p_T(v_1)$ and $p_T(v_2)$ are measured in the laboratory transverse plane then ϕ_{12} is the angle between v_1 and v_2 in that plane.

It is interesting to note at this point that when v_1 and v_2 are massless and the $\delta_1\delta_2$ CoM frame is at rest in the laboratory transverse plane the $\mathbf{E_T}^{\mathrm{miss}}$ vector can be represented under a change of basis involving $M_{\mathrm{CT}}(v_1, v_2)$:

$$\mathbf{E_T}^{\mathrm{miss}} = \{-p_x(v_1) - p_x(v_2), -p_y(v_1) - p_y(v_2)\} \rightarrow \{p_T(v_1) - p_T(v_2), M_{\mathrm{CT}}(v_1, v_2)\}. \quad (2.16)$$

In the new basis the first component can be interpreted as the contribution to E_T^{miss} from p_T asymmetry, while the second, containing the geometric mean of $p_T(v_1)$ and $p_T(v_2)$, can be interpreted as the contribution from event topology. In this case E_T^{miss} is given by:

$$E_T^{\mathrm{miss}} = \sqrt{[p_T(v_1) - p_T(v_2)]^2 + M_{\mathrm{CT}}^2(v_1, v_2)}. \qquad (2.17)$$

The physical interpretation of the contransverse mass is more difficult than in the (co)transverse case. $M_{\mathrm{CT}}(v_1, v_2)$ does not represent the mass of a particle decaying to produce v_1 and v_2. Nevertheless we expect its distribution to display an end-point because it can in principle be calculated from the momenta of visible decay products measured in the rest frames of δ_1 and δ_2, and we know from section 2.2 that these momenta are

bounded from above by $\mathcal{M}_i/2$. For instance if $m(v_1) = m(v_2) = 0$ then $M_{CT}(v_1, v_2)$ takes a maximum value of $\mathcal{M}_0$, i.e.

$$M_{CT}^{max} = \frac{m^2(\delta) - m^2(\alpha)}{m(\delta)}. \tag{2.18}$$

Interestingly this bound is saturated when v_1 and v_2 are co-linear, in contrast to the case for the (co)transverse mass.

To summarise, we have now found a quantity bounded from above by an analytical combination of particle masses, and which can be calculated using momenta of visible decay products measured in the laboratory transverse plane. We shall now consider as a use-case the practical application of this variable to LHC data in order to measure SUSY particle masses.

3. Example: $\tilde{q}_R\tilde{q}_R$ events at the LHC

To illustrate the application of the contransverse mass end-point technique to LHC data, a Monte Carlo simulation study was carried out aimed at measuring M_{CT}^{max} for $\tilde{q}_R$ pair production events where each $\tilde{q}_R$ decays to a quark and a $\tilde{\chi}_1^0$. Squark mass measurement in this channel using the stransverse mass method was first studied in ref. [15]. The experimental signature of this process is the presence of events with exactly two jets and large E_T^{miss}. In the context of the decay chain discussed in section 2.1 the $\tilde{q}_R$ plays the role of δ and $\tilde{\chi}_1^0$ that of α. We assume that the quark jet decay products are massless and hence eq. 2.18 allows us to measure an analytical combination of sparticle masses by measuring M_{CT}^{max}.

A sample of 480k SUSY signal events equivalent to $10\,\text{fb}^{-1}$ of data was generated from the SPS1a benchmark mSUGRA model [15] with `HERWIG 6.5` [16, 17] and passed to a generic LHC detector simulation [18] modified to impose an 80% efficiency for electron identification, with mis-identified electrons being added to the list of jets if $p_T(e) > 10\,\text{GeV}$. The `ISASUGRA 7.69` RGE code [19] was used to calculate the input SUSY mass spectrum, giving $m(\tilde{q}_R) \sim 548\,\text{GeV}$ and $m(\tilde{\chi}_1^0) = 96\,\text{GeV}$ and hence $M_{CT}^{max} = 531\,\text{GeV}$. A fully inclusive sample of SUSY events was generated in order to model SUSY backgrounds.

Events were selected with the following requirements (with j_i used to denote jet i):

- $n_{jet} = 2$ for $\Delta R = 0.4$ cone jets with $p_T(j) > 10\,\text{GeV}$ and $|\eta| < 5.0$,

- $n_{lep} = 0$ for isolated leptons (electrons or muons) with $p_T > 5\,\text{GeV}$ (electrons) or $p_T > 6\,\text{GeV}$ (electrons), $|\eta| < 2.5$, minimum ΔR with nearest jet of 0.4 and maximum energy deposition of $10\,\text{GeV}$ in a $\Delta R = 0.2$ isolation cone,

- $\min[p_T(j_1), p_T(j_2)] > 100\,\text{GeV}$,

- $E_T^{miss} > 200\,\text{GeV}$,

- in order to limit boosts of the $\tilde{q}_R\tilde{q}_R$ CoM frame in the laboratory transverse plane, measured p_T of the $j_1 j_2 + E_T^{miss}$ CoM frame in the laboratory transverse plane must

satisfy

$$\sqrt{[p_x(j_1) + p_x(j_2) + E_x^{\mathrm{miss}}]^2 + [p_y(j_1) + p_y(j_2) + E_y^{\mathrm{miss}}]^2} < 20\,\mathrm{GeV}, \qquad (3.1)$$

- $M_{\mathrm{CT}} > 200\,\mathrm{GeV}$.

The hard $p_T(j)$ and E_T^{miss} cuts additionally ensure that events easily pass typical LHC high level jet + E_T^{miss} trigger criteria such as $p_T(j) > 70\,\mathrm{GeV}$ and $E_T^{\mathrm{miss}} > 70\,\mathrm{GeV}$ [20].

After application of these cuts many Standard Model (SM) backgrounds are heavily suppressed:

- QCD jet backgrounds, while possessing a very large cross-section, are suppressed by the M_{CT} cut which rejects events with back-to-back jets. Jet energy mis-measurement mainly generates E_T^{miss} through the first term in eq. 2.17 and the effect on M_{CT} is smaller. In order to pass the M_{CT} cut at least one high p_T jet must be completely missed by the detector. The M_{CT} cut is strongly correlated with the $D_{\pi\pi}$ variable used at the Tevatron to separate SUSY signal from QCD backgrounds in multijet+E_T^{miss} searches [21]. The fast detector simulation used in this study is not expected to model the catastrophic loss of jets accurately, but we do not expect this background to be dominant even when using a more realistic simulation. In particular such events can in principle be removed with 'event cleaning' cuts, for instance by reconstructing jets from charged particle tracks. Consequently QCD jet backgrounds are not considered further here.

- Hadronic or semi-leptonic $t\bar{t}$ backgrounds are suppressed by the jet multiplicity cuts, while fully leptonic events in which both leptons are lost inside the jets (the worst case scenario kinematically) possess M_{CT} values less than $m(t) \sim 172\,\mathrm{GeV}$, which is the value expected for top quarks decaying to a neutrino plus massless visible decay products. Such events therefore fail the M_{CT} cut.

- $W + 1$ jet backgrounds in which the W decays to a hadronic τ or electron faking a jet can mimic 2-jet events. Events with large M_{CT} typically require two co-linear jets of similar $p_T(j)$. Kinematically such events must possess $p_T(j)$ values lying below

$$p_T^{\max}(j) = \gamma\beta m(W) = \frac{1}{2}m(W)\gamma(1 - \beta), \qquad (3.2)$$

where β is the boost of the W in the transverse plane. This equation can be solved to yield $p_T^{\max}(j) = m(W)/2\sqrt{2} \sim 29\,\mathrm{GeV}$ and hence $M_{\mathrm{CT}}^{\max} = 2p_T(j) = 58\,\mathrm{GeV}$. Such events therefore also fail the M_{CT} cut and are not considered further here.

The remaining SM backgrounds are dominated by $Z(\to \nu\nu) + 2$ jets and $W(\to l\nu) + 2$ jets events, where in the latter case the lepton momentum is anti-parallel to the W momentum and is 'red-shifted' such that its magnitude is below the lepton identification p_T threshold. These backgrounds were modelled with `ALPGEN` [22] coupled to `HERWIG 6.5`. In order to add realism to the analysis the backgrounds were estimated using data-driven

techniques applied to the Monte Carlo 'data'. The $Z(\to \nu\nu) + 2$ jets background was estimated by selecting $Z(\to ll) + 2$ jets events with similar cuts to those listed above, but replacing the lepton veto and E_T^{miss} requirements with a requirement for two opposite-sign same-flavour leptons with $|m(ll) - m(Z)| < 10\,\mathrm{GeV}$ and $|\mathbf{p_T}(ll) + \mathbf{E_T}^{\mathrm{miss}}| > 200\,\mathrm{GeV}$. The $W(\to l\nu) + 2$ jets background was estimated by selecting $W(\to l\nu) + 2$ jets events in which the lepton was boosted and the neutrino de-boosted such that $p_T(l) > 200\,\mathrm{GeV}$ and $E_T^{\mathrm{miss}} < 10\,\mathrm{GeV}$. Each data-driven estimate was normalised separately to the respective Monte Carlo background $p_T(j)$ distribution below the expected SUSY signal region. In practice the relative normalisation of the $W(\to l\nu) + 2$ jets estimate could be obtained from data, for instance with a fit to the lepton p_T spectrum in $W(\to l\nu) + 2$ jets events.

SUSY backgrounds to $\tilde{q}_R\tilde{q}_R$ events arise primarily from processes in which at least one $\tilde{q}_L$ decays through a chain producing multiple invisible final state particles. One possible example involves sneutrinos decaying to neutrinos and $\tilde{\chi}_1^0$. In these cases the mass of each SUSY state produced in association with the jet in the decay of each $\tilde{q}_L$ is greater than that of the $\tilde{\chi}_1^0$ produced in the decay of $\tilde{q}_R$ and consequently these events possess $M_{\mathrm{CT}}^{\mathrm{max}}$ values below those for $\tilde{q}_R\tilde{q}_R$. At parton level if SUSY background events are to exceed the expected end-point, assuming correct assignment of decay products to SUSY decay chains, then the mass of the initially produced sparticles must be greater than $m(\tilde{q}_R)$. The main candidate for this is $\tilde{g}$ pair production in which each gluino decays to co-linear jets in association with a $\tilde{\chi}_1^0$. This process should generate a M_{CT} distribution with an endpoint at $M_{\mathrm{CT}}^{\mathrm{max}} = 597\,\mathrm{GeV}$ for a $\tilde{g}$ mass of $612\,\mathrm{GeV}$ at SPS1a.

The M_{CT} distribution for events satisfying the selection cuts is shown in figure 1(left) indicating an excess of events at large M_{CT} values due to SUSY processes. As expected the contribution from $t\bar{t}$ events (modeled with `HERWIG 6.5`) is small, as are the contributions from WW, WZ, ZZ and single-top production (also modeled with `HERWIG 6.5`). The data-points in figure 1(right) represent the same distribution after subtracting the data-driven background estimate. As expected, a prominent end-point feature is visible at around $530\,\mathrm{GeV}$. A simple linear fit to the endpoint determines its position to be $550 \pm 53\,\mathrm{GeV}$ (10% uncertainty). Use of a more sophisticated fitting function would undoubtedly improve this precision significantly. There is also some evidence in figure 1(right) for a small excess of events beyond the expected $\tilde{q}_R\tilde{q}_R$ end-point. Examination of the Monte Carlo truth record indicates that the large M_{CT} values of these events originate either from jet mis-measurement in $\tilde{q}_R\tilde{q}_R$ events or from both jets originating from the same SUSY decay chain in non-$\tilde{q}_R\tilde{q}_R$ events. No $\tilde{g}\tilde{g}$ events were observed to contribute to this region for this SPS1a SUSY model.

In order to study the dependence of the shape and position of the M_{CT} end-point in figure 1 on the cut on the measured p_T of the $j_1 j_2 + E_T^{\mathrm{miss}}$ CoM frame (eq. 3.1), the M_{CT} distributions of SUSY signal events passing progressively harder p_T cuts were generated. Due to the strong correlation between the di-jet multiplicity cut and the p_T cut the former cut was relaxed to require *at least* two jets, with the two hardest jets being used to calculate M_{CT}. The resulting distributions are plotted in figure 2 for ten different values of the p_T cut ranging from $200\,\mathrm{GeV}$ (top) to $20\,\mathrm{GeV}$ (bottom). The effect of the cut on the SUSY signal is to sharpen the end-point at the expense of statistics. This sharpening of the
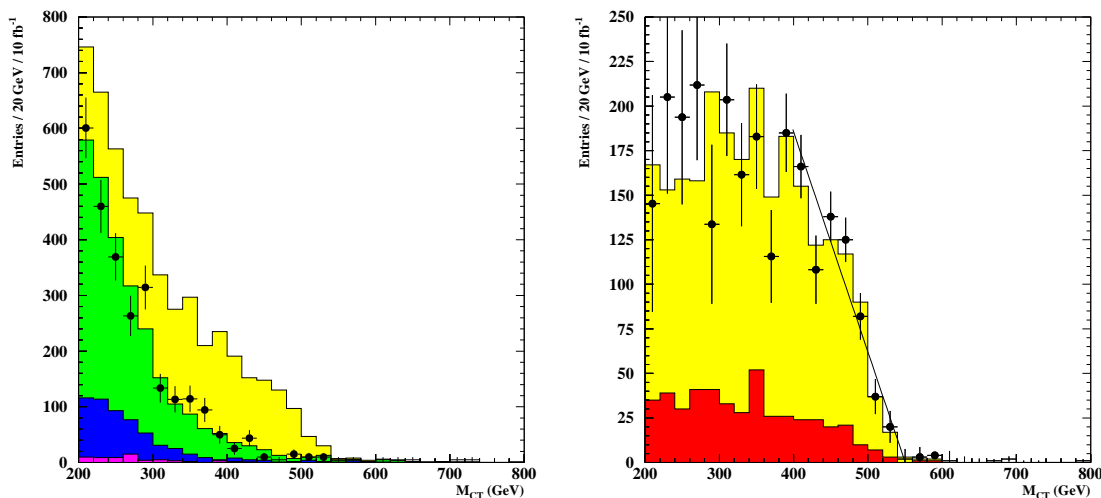
Figure 1: Distributions of $M_{\rm CT}$ values. The left-hand figure shows the cumulative 'data' distributions summing SPS1a SUSY events (light/yellow histogram), $Z(\to \nu\nu) + 2$ jets background events (medium/green), $W(\to l\nu)+2$ jets background events (dark/blue) and diboson, top-pair and single-top background events (medium-dark/magenta). Data-points indicate the result of the data-driven estimate described in the text. The data-points in the figure on the right show the result of subtracting the data-driven estimate from the 'data' distribution, with the light/yellow histogram representing the SUSY distribution with no SM background added. The dark/red histogram shows the contribution from non-$\tilde{q}_R\tilde{q}_R$ SUSY events. The result of a simple linear end-point fit to the data-points is shown.

end-point is caused both by limitation of the transverse boost of the $\tilde{q}_R\tilde{q}_R$ CoM frame and by rejection of high multiplicity non-$\tilde{q}_R\tilde{q}_R$ events in which two jets from the same SUSY decay chain are selected to calculate $M_{\rm CT}$. It should also be noted that a harder p_T cut rejects more SM background events, especially QCD multijet events.

4. Extension to measurement of individual particle masses

So far we have shown that we can obtain an end-point in the distribution of a quantity calculated from visible decay product transverse momenta which depends on an analytical combination of masses. In principle it is possible to use the position of this end-point to measure the individual masses $m(\delta)$ and $m(\alpha)$. The key to this is recognising that $\mathcal{M}_i$ depends upon both the unknown masses $m(\delta)$ and $m(\alpha)$ and the visible masses $m(v_i)$. If one requires that $m(v_1) = m(v_2) = m(v)$ then it can be shown from eqs. 2.4 and 2.14 that for given $m(v)$ the bound on $M_{\rm CT}$ is given by:

$$M_{\rm CT}^{\max} = \frac{1}{m(\delta)}m^2(v) + \mathcal{M}_0. \tag{4.1}$$

Consequently if events could be found in which $m(v_1)$ and $m(v_2)$ were non-zero and equal, for instance by accurately combining products from several decays in a multi-step
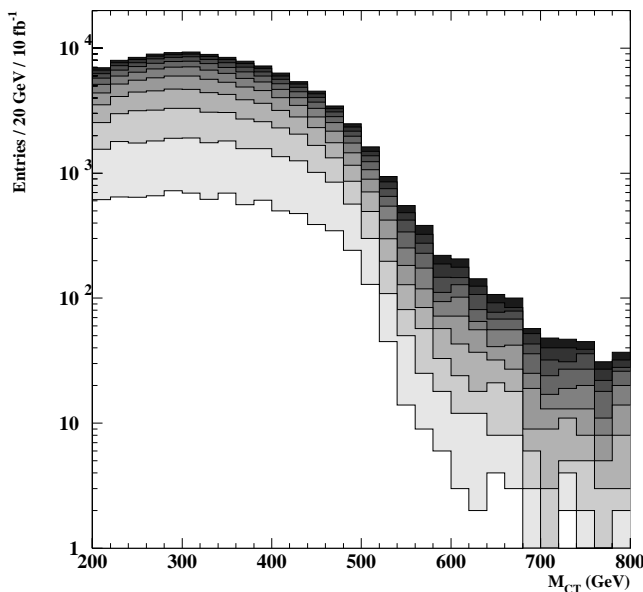
Figure 2: Distributions of M_{CT} values for SUSY signal events for ten different values of the cut on the measured p_T of the $j_1 j_2 + E_T^{\mathrm{miss}}$ CoM frame (eq. 3.1), ranging from $200\,\mathrm{GeV}$ (top) to $20\,\mathrm{GeV}$ (bottom) in $20\,\mathrm{GeV}$ steps. In contrast to figure 1 the jet multiplicity cut has been relaxed to require at least two jets and a logarithmic y-axis has been used to aid comparison of shapes of distributions.

chain, then the position of $M_{\mathrm{CT}}^{\max}$ would depend linearly on $m^2(v)$ with a gradient of $1/m(\delta)$ and intercept the ordinate at $\mathcal{M}_0$. The gradient of a linear fit to this bound therefore measures $m(\delta)$ independently of $m(\alpha)$, while the intercept then allows $m(\alpha)$ to be constrained. As an aside it is interesting to note that in this case M_{CT} is also bounded from below by:

$$M_{\mathrm{CT}}^{\min} = 2\sqrt{m^2(v)}. \tag{4.2}$$

and for exclusive decay chains $m^2(v)$ is bounded from above by a separate analytical combination of masses identical to that used by the end-point method discussed in section 1.

The above technique should work in principle however it is likely to be very difficult to implement in practice. The first difficulty is connected with unambiguously associating decay products with SUSY decay chains. One possible approach would involve focusing on specific exclusive decay chains, using the values of invariant masses of combinations of decay products to associate decay products to chains [4]. Unfortunately however the low acceptance of such exclusive selections is likely to prevent successful application of this technique before significant quantities of data have been acquired.

A second approach involves inclusive selection of SUSY events with multiple visible decay products, and use of a kinematic algorithm to approximately associate decay products to chains. Figure 3(left) shows the result of an attempt at applying this approach to SPS1a
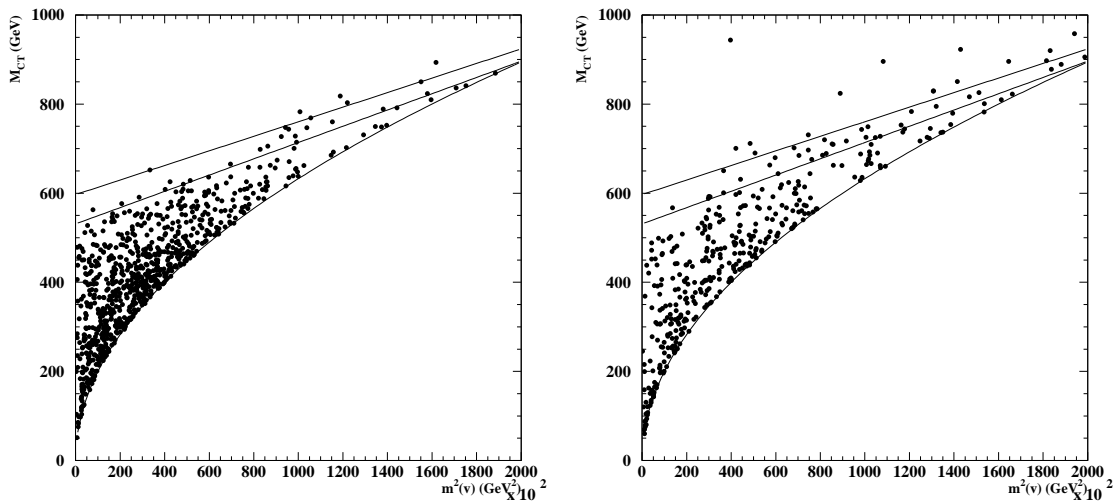
Figure 3: Distribution in the $M_{\rm CT} - m^2(v)$ plane of SPS1a events with ISR turned off (left) and on (right). The top (straight) line represents the expected dependence from eq. 4.1 of $M_{\rm CT}^{\max}$ on $m^2(v)$ for events containing $\tilde{g}$ pair production. The middle (straight) line represents the equivalent dependence for events containing $\tilde{q}$ pair production. The bottom line represents the expected dependence of $M_{\rm CT}^{\min}$ on $m^2(v)$ given by eq. 4.2.

events *with ISR turned off*. Here decay products have been associated to chains by requiring that $\max[m^2(v_1), m^2(v_2)]$ is minimised. Events have been selected by requiring four jets and no leptons, and the mass equality requirement mentioned above has been imposed by requiring that the asymmetry in $m^2(v_1)$ and $m^2(v_2)$ is less than 10%. An additional cut requiring the rapidity difference between v_1 and v_2 to be greater than 1.0 has also been applied to reduce combinatorics.

Figure 3(left) shows that with these cuts events generally lie below the expected upper bounds on $M_{\rm CT}$ for $\tilde{g}$ decays (top line) or $\tilde{q}$ decays (middle line), although some combinatorial contamination is visible above the $\tilde{g}$ bound. The lower bound on $M_{\rm CT}$ given by eq. 4.2 is prominent (bottom line). Figure 3(right), obtained with SPS1a events *with ISR turned on*, illustrates the further difficulty of using this approach however. The inclusion of ISR jets in v_1 and/or v_2 can artificially lower $m^2(v)$ and hence generate false configurations which strongly violate the upper bounds on $M_{\rm CT}$. A related effect was noted previously in connection with the stransverse mass technique in ref. [8]. Clearly much more work is needed before this technique can be used practically to measure accurately independent particle masses.

5. Conclusions and directions for future work

This paper has shown that by constructing a kinematic quantity invariant under contra-linear equal magnitude boosts in the laboratory transverse plane a simple analytical com-

bination of the masses of pair-produced particles and their invisible decay products can be constrained at hadron colliders such as the LHC. It was shown that in principle these techniques may be used to measure the masses of such particles independently, although in practice this seems to be very difficult.

The study described in this paper suggests several directions for future work. These include:

- The experimental simulation study of section 3 should be repeated with more realistic full experiment-specific simulation of all Standard Model and SUSY backgrounds to demonstrate conclusively the feasibility of these techniques when applied to $\tilde{q}_R\tilde{q}_R$ events. The feasibility of $\tilde{l}$ mass measurement with $\tilde{l}\tilde{l}$ events should also be studied.

- Further work assessing the feasibility of measuring independent particle masses using the technique outlined in section 4 is required, focusing in particular on optimising the experimental assignment of decay products to decay chains, and rejection of ISR jets.

Acknowledgments

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